It thus seems appropriate to interpret our numerical prediction of separation less than one cell below the corner for $Re \geq 25$ as separation at the corner itself within the accuracy of the finite-difference calculation.

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Technical Comments

Further Comments on "Local Nonsimilarity Boundary-Layer Solutions"

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SPARROW, Quack, and Boerner[1] have developed a new method of solution for nonsimilar boundary layers and presented results for several representative problems. Coxon and Parks[2] commented that Sparrow, Quack, and Boerner neglected a term on the right-hand side of the first auxiliary momentum equation which could be included. Coxon and Parks expressed interest in the effect of retaining this extra term on the accuracy of the results. The results presented below will show that for a particular case the effect is adverse, i.e., the accuracy is decreased. Thus, the closure condition advanced by Sparrow[3] in his reply to Coxon and Parks is to be preferred.

As part of an ongoing study of compressible similar and nonsimilar laminar boundary layers, the effect of this choice of closure condition for Howarth's incompressible retarded flow was investigated. For Howarth's flow, the freestream velocity is given by

$$U(x)/U_\infty = 1-(x/L) \tag{1}$$

Following Ref. 1, the transformations

$$\xi = x/L, \quad \eta = y(U/\nu x)^{1/2}, \quad \psi = (\nu U x)^{1/2} f(\xi,\eta) \tag{2}$$

when applied to the incompressible steady boundary layer equations and no mass transfer boundary conditions, yield

$$f''' + [(\Omega+1)/2]ff'' + \Omega(1-f'^2) = \xi(f'g'-f''g) \tag{3}$$

with

$$f(\xi,0) = f'(\xi,0) = 0, \quad f'(\xi,\eta\to\infty)\to 1 \tag{4}$$

where

$$\Omega = (x/U)(dU/dx) = \xi/(\xi-1) \tag{5}$$

and the function

$$g(\xi,\eta) = \partial f/\partial \xi \tag{6}$$

The first auxiliary momentum equation for the locally nonsimilar method is obtained by differentiatng Eqs. (3) and (4). This yields

$$g''' + [(\Omega+1)/2][gf''+fg''] + (1/2)(d\Omega/d\xi)ff'' + (d\Omega/d\xi)(1-f'^2) - 2\Omega f'g' = (f'g'-f''g) + \xi(\partial/\partial\xi)(f'g'-f''g) \tag{7}$$

with boundary conditions

$$g(\xi,0) = g'(\xi,0) = 0; \quad g'(\xi,\eta\to\infty)\to 0 \tag{8}$$

For the two-equation locally nonsimilar method, the closure condition used in Ref. 1 and recommended in Ref. 3 neglects the entire term $\xi\partial/\partial\xi(f'g'-f''g)$. However, this term may be expanded to yield $\xi(g'^2-g''g)+\xi(f'g'_\xi-f''g_\xi)$. Thus Eq. (1) then may be written as

$$g''' + [(\Omega+1)/2](gf''+fg'') + (1/2)(d\Omega/d\xi)ff'' + (d\Omega/d\xi)(1-f'^2) - 2\Omega f'g' = (f'g'-f''g) + \xi(g'^2-g''g) + \xi(f'g_\xi'-f''g_\xi) \tag{9}$$

Examination of Eqs. (3) and (9) with boundary conditions given by Eqs. (4) and (8) shows that neglecting only the last term on the right-hand side of Eq. (9) will yield a system of ordinary differential equations at each streamwise location ξ. This is the alternate closure condition discussed by Coxon and Parks.

The effect of the choice of closure condition, i.e., including or not including the term $\xi(g'^2-g''g)$, was investigated by integrating both sets of equations using a fourth-order Runge-Kutta integration scheme with a fixed step size of 0.01.

Table 1 Comparison of closure conditions for the LNS method applied to Howarth flow[a]

ξ	Without term $\xi(g'^2-g''g)$ $f''(0)$	Without term $\xi(g'^2-g''g)$ $g''(0)$	With term $\xi(g'^2-g''g)$ $f''(0)$	With term $\xi(g'^2-g''g)$ $g''(0)$
0.0	0.33205754	−1.5429949	0.33205754	−1.5429949
0.01	0.31635425	−1.6254251	0.31656385	−1.6475604
0.02	0.30008091	−1.7167614	0.30122970	−1.7707965
0.03	0.28324570	−1.8185079	0.28679669	−1.9175033
0.04	0.26584963	−1.9324951	0.27463834	−2.0914424
0.05	0.24792076	−2.0609066	0.26688397	−2.2899082
0.06	0.22951139	−2.2063531	0.26560624	−2.4970761
0.07	0.21070638	−2.3719145	0.27115886	−2.6892762
0.075	0.20119169	−2.4633480	0.27605213	−2.7744743
0.076	...	...	0.27716689	−2.7905409
0.08	0.19162534	−2.5612217	No solutions	
0.09	0.17240954	−2.7786882	No solutions	
0.095	0.16278799	−2.8997843	No solutions	
	No solutions			

[a] Numerical results are given to eight significant figures for information purposes only. Results are considered accurate to $\pm 5\times10^{-6}$. $\eta_{max} = 12.0$ for all calculations.

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Index category: Boundary Layers and Convective Heat Transfer—Laminar.

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Satisfaction of the asymptotic boundary conditions was achieved by using a modified Nachtscheim-Swigert iteration scheme.[4] Asymptotic boundary conditions were considered satisfied when they were within $\pm 5 \times 10^{-6}$ of the required value. The results are shown in Table 1. Unfortunately, numerical values were not presented in Ref. 1; thus a detailed comparison is not possible. However, comparison with Fig. 1 of Ref. 1 indicates general agreement with the present results when the $\xi(g'^2 - g''g)$ term is neglected.

Examination of Table 1 shows that solutions with the term $\xi(g'^2 - g''g)$ could be obtained to the required accuracy only for $\xi \leqq 0.076$. Although a number of techniques were used, no solutions could be obtained for values of $\xi > 0.076$ when the term $\xi(g'^2 - gg'')$ was included. This indicates a mathematical instability which yields results that are not physically reasonable. Furthermore, detailed examination of the values of $f''(0)$, the nondimensional shearing stress, for $\xi = 0.070$, 0.075, and 0.076, shows that $f''(0)$ is increasing. The results in Table 1 also show that acceptable solutions without the term $\xi(g'^2 - gg'')$ could be obtained only for values of $\xi \leqq 0.95$. Note that this is approximately the limit of ξ shown in Fig. 1 of Ref. 1. Beyond $\xi = 0.095$, attempts to obtain solutions again indicate that $f''(0)$ increases.

The results of the present numerical experiment indicate that the original closure condition used in Ref. 1 is to be preferred.

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Errata

Ignition Analysis of Adiabatic, Homogeneous Systems Including Reactant Consumption

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[AIAA J. **11**, 1728–1731 (1973)]

EQUATION (11) should read:

$$\tau = u + \varepsilon\{[(\gamma+2)[u+(1-u)\ln(1-u)] - (1-u)\ln^2(1-u)\} + \varepsilon^2\{2Au + (1-u)[2A\ln(1-u) - A\ln^2(1-u) + B\ln^3(1-u) - \tfrac{1}{2}\ln^4(1-u)]\}$$

where A and B are as given.

The fourth line, second column, on p. 1731 of the Appendix should read in part:

"...the higher-order solutions, F_i, $i \neq 0$ should be (no) more complicated..." The "no" was added.

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Nonlinear Vibration of Orthotropic Triangular Plates

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[AIAA J. **11**, 704–709 (1973)]

IN Table 1 on p. 706, the shear modulus G_{12} of the isotropic material should be read as 0.385×10^5 ksc. The expressions on the right-hand sides of the two equations immediately after Eq. (26) on p. 708 should be replaced by $0.27063 \times 10^{-2}[c(K_{12} - K_3)/b]\tau^2$ in the first equation and $0.27063 \times 10^{-2}[(3c^2K_2 - K_3 - 2K_{12})/b]\tau^2$ in the second equation.

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